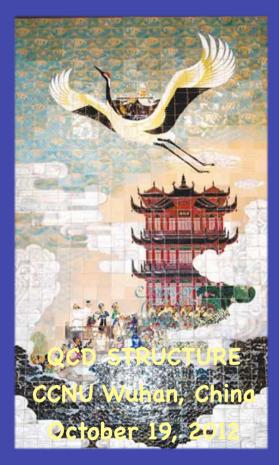
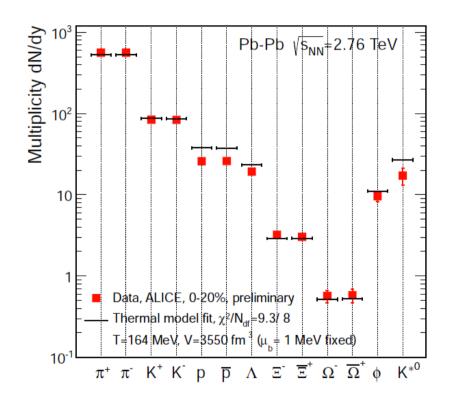
QCD and high energy density Thermalization of the QGP

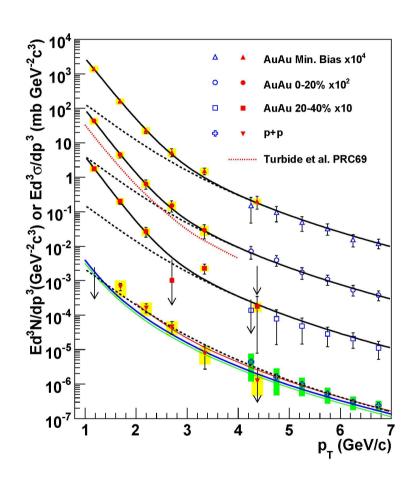




Some evidence for thermal behavior



Chemical equilibrium at freeze-out

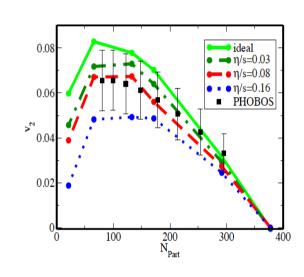


Thermal photons (PHENIX)

Hydrodynamical flow

- Matter produced in heavy ion collisions exhibits fluid behavior from very early time on (elliptic flow, sensitivity to fluctuations in initial conditions, etc)
- The fluid has very special transport properties, in particular a small value of the shear viscosity to entropy density ratio

viscosity?



Fluid behavior requires (some degree of) local equilibration. How is this achieved? How can we understand the small (relative) value of the

A theoretical puzzle

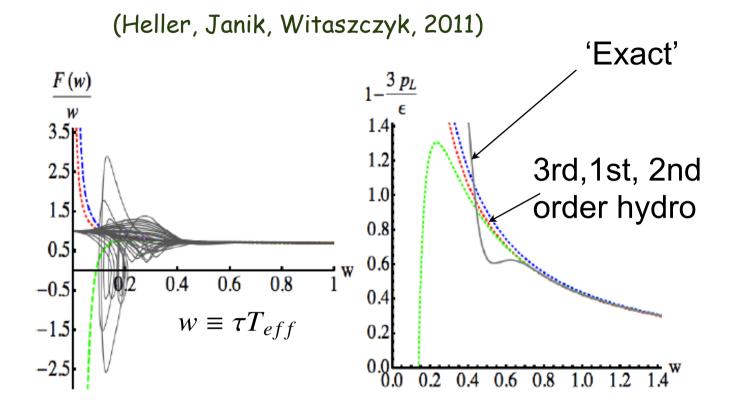
Small η/s and short equilibration time seem incompatible with weak coupling

However, the coupling constant is not very large $\alpha_{\rm s} \sim 0.3 \div 0.4$

Strong coupling conflicts with our understanding of initial stages of heavy ion collisions, which is based on weak coupling (for asymptotically large nuclei and large energies)

 $Q_s^2 \approx \alpha_s \frac{xG(x,Q^2)}{\pi R^2}$

Holographic description of a boost invariant plasma



Viscous hydro can cope with partial thermalization, and large differences between longitudinal and transverse pressures

In fact, there is little experimental evidence that complete local equilibrium is reached in nuclear collisions

What is the fluid made of?

What are the important degrees of freedom?

(quasi) particles? massive quarks and gluons?

(classical color) fields?

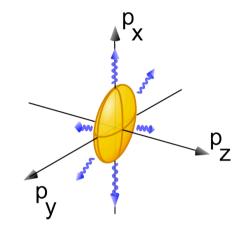
or both?

In a plasma, at weak coupling, there is a clear separation of hard (particles) and soft (collective) modes, that are coupled together (hard loops)

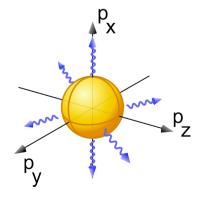
$$f(\mathbf{p}, X) = f_0(\mathbf{p}) + \delta f(\mathbf{p}, X)$$
$$[v \cdot D_X, f(\mathbf{p}, X)] + g v_{\mu} F^{\mu\nu} \partial_{\nu}^{(p)} f_0(\mathbf{p}) = 0$$
$$D_{\mu} F^{\mu\nu} = J^{\nu} = g \int_{p} v^{\nu} \delta f(\mathbf{p}, X)$$

Mean field instabilities Isotropization

Collective modes in anisotropic plasmas can become instable for some range of momenta (e.g. Weibel instability)



Instabilities contribute to restore/maintain isotropy

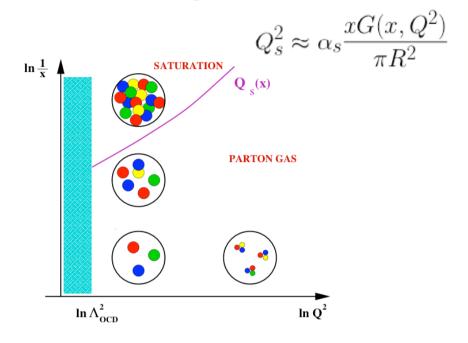


The over-populated quark-gluon plasma

High density partonic systems

Large occupation numbers

$$\frac{xG(x,Q^2)}{\pi R^2 Q_s^2} \sim \frac{1}{\alpha_s}$$



saturation fixes the initial scale

$$\epsilon_0 = \epsilon(\tau = Q_{\rm s}^{-1}) \sim \frac{Q_{\rm s}^4}{\alpha_{\rm s}}$$
 $n_0 = n(\tau = Q_{\rm s}^{-1}) \sim \frac{Q_{\rm s}^3}{\alpha_{\rm s}}$ $\epsilon_0/n_0 \sim Q_{\rm s}$

Thermodynamical considerations

Initial conditions $(t_0 \sim 1/Q_s)$

$$\epsilon_0 = \epsilon(\tau = Q_{\rm s}^{-1}) \sim \frac{Q_{\rm s}^4}{\alpha_{\rm s}}$$
 $n_0 = n(\tau = Q_{\rm s}^{-1}) \sim \frac{Q_{\rm s}^3}{\alpha_{\rm s}}$ $\epsilon_0/n_0 \sim Q_{\rm s}$

overpopulation parameter

$$n_0 \epsilon_0^{-3/4} \sim 1/\alpha_{\rm s}^{1/4}$$

in equilibrated quark-gluon plasma

$$\epsilon_{\rm eq} \sim T^4$$

$$n_{\rm eq} \sim T^3$$

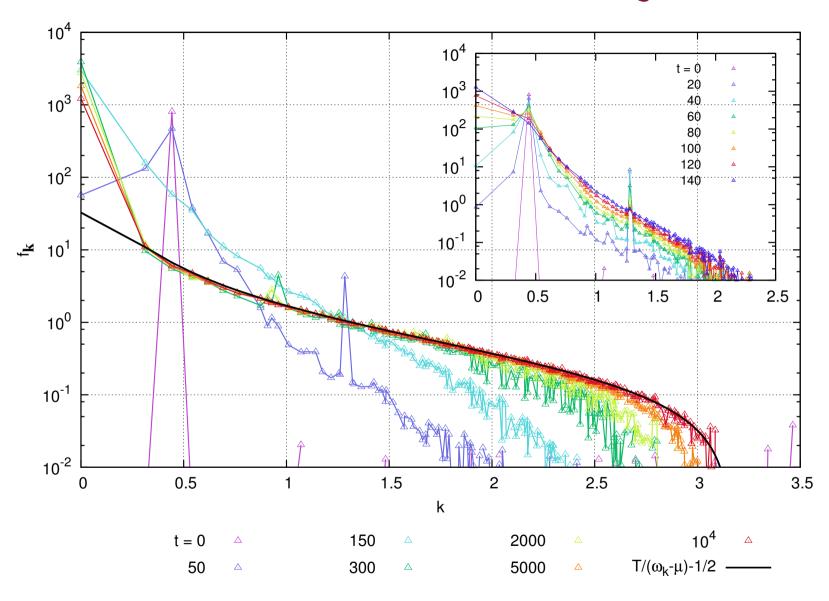
$$n_{\rm eq} \, \epsilon_{\rm eq}^{-3/4} \sim 1$$

mismatch by a large factor (at weak coupling) $~\alpha_{
m s}^{-1/4}$

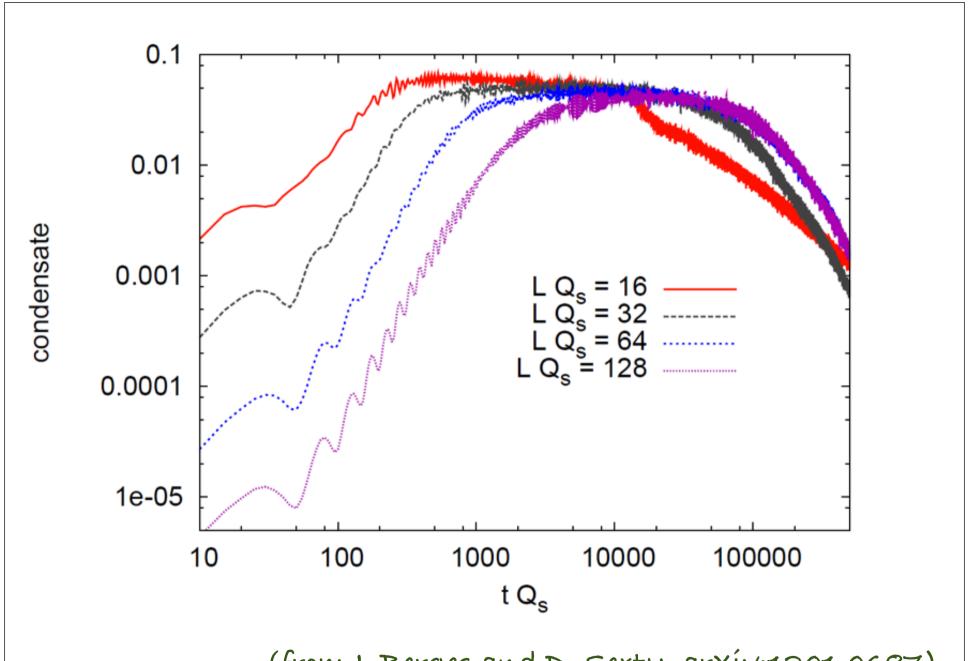
Will the system accommodate the particle excess by forming a Bose-Einstein condensate?

(JPB, F. Gelis, J. Liao, L. McLerran, R. Venugopalan, 2012)





(T. Epelbaum and F. Gelis, 2011)



(from J. Berges and D. Sexty, arxiv:1201.0687)

Kinetic evolution dominated by elastic collisions

[work in progress with Jinfeng Liao and Larry McLerran]

Boltzmann equation with 2->2 scattering

Gluon distribution function

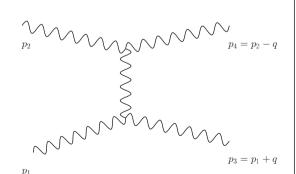
$$f(\mathbf{x}, \mathbf{p}) = \frac{(2\pi)^3}{2(N_c^2 - 1)} \frac{dN}{d^3 \mathbf{x} d^3 \mathbf{p}}$$

Boltzmann equation

$$\mathcal{D}_t f_1 = \frac{1}{2} \int \frac{d^3 p_2}{(2\pi)^3 2E_2} \frac{d^3 p_3}{(2\pi)^3 2E_3} \frac{d^3 p_4}{(2\pi)^3 2E_4} \frac{1}{2E_1} |M_{12\to 34}|^2 \times (2\pi)^4 \delta(p_1 + p_2 - p_3 - p_4) \{f_3 f_4 (1 + f_1)(1 + f_2) - f_1 f_2 (1 + f_3)(1 + f_4)\}$$

$$\mathcal{D}_t \equiv \partial_t + \vec{v}_1 \cdot \vec{\nabla}$$

$$|M|^2 = 576g^4 E_1^2 E_2^2 \left| \frac{1 - x^2}{\boldsymbol{q}_0^2 - \boldsymbol{q}^2 - \Pi_L} - \frac{(1 - x^2)\cos\phi_2}{q_0^2 - \boldsymbol{q}^2 - \Pi_T} \right|^2$$



Small angle approximation

$$\mathcal{D}_t f = C(f) \qquad C(f) = -\nabla \cdot \mathbf{S} = -\frac{\partial S_i}{\partial p_i}$$

Simplified kinetic equation

$$\mathcal{D}_{\tau}f(\boldsymbol{p}) = \boldsymbol{\nabla} \cdot \left[I_a \boldsymbol{\nabla} f(\boldsymbol{p}) + \frac{\boldsymbol{p}}{p} I_b f(\boldsymbol{p}) [1 + f(\boldsymbol{p})] \right]$$

$$I_a = \int \frac{d^3p}{(2\pi)^3} f(\boldsymbol{p}) (1 + f(\boldsymbol{p}))$$

$$I_b = \int \frac{d^3p}{(2\pi)^3} \frac{2f(\mathbf{p})}{p}$$

'universal' equation

$$\tau = 36\pi\alpha^2 \mathcal{L}t \qquad \mathcal{L} = \int \frac{dq}{q}$$

Note: when $f \sim 1/\alpha_{\rm s}$ all dependence on coupling disappears --> classical field dynamics

Some results

Solve

$$\mathcal{D}_{\tau}f(p) = I_a \frac{\partial^2 f}{\partial p^2} + I_b (1 + 2f) \frac{\partial f}{\partial p} + \frac{2}{p} \left[I_a \frac{\partial f}{\partial p} + I_b f (1 + f) \right]$$

with initial condition

$$f(p) = f_0 \theta(Q_s - p)$$
 $\epsilon_0 = f_0 \frac{Q_s^4}{8\pi^2}$ $n_0 = f_0 \frac{Q_s^3}{6\pi^2}$

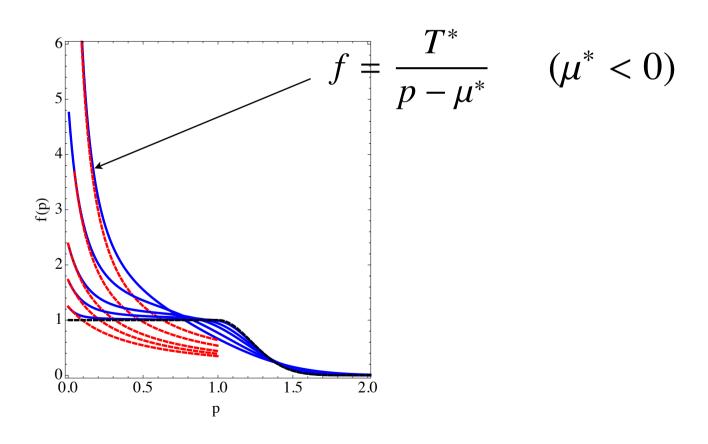
Onset of BEC

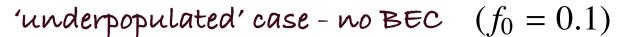
$$n_0 \,\epsilon_0^{-3/4} = f_0^{1/4} \, \frac{2^{5/4}}{3 \,\pi^{1/2}} \qquad n \,\epsilon^{-3/4}|_{SB} = \frac{30^{3/4} \,\zeta(3)}{\pi^{7/2}} \approx 0.28.$$

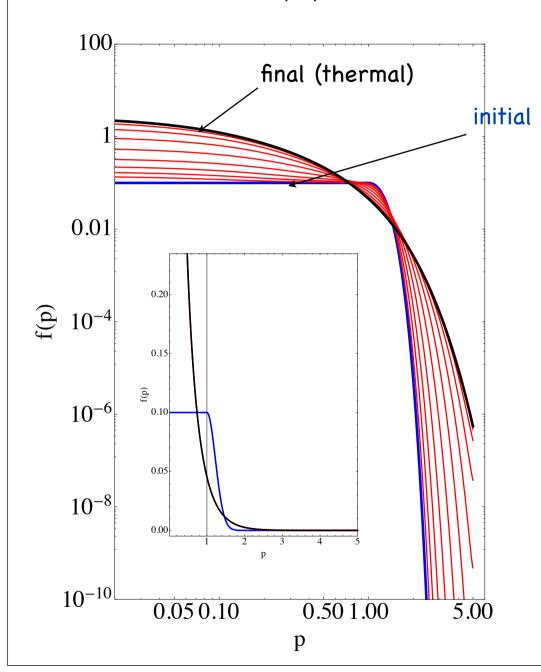
$$f_0^c \approx 0.154$$

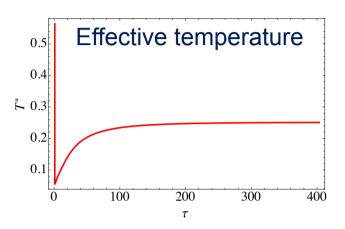
Small momentum behavior

At small momentum $(f\gg 1)$ the distribution quickly converges towards an 'instantaneous' equilibrium distribution function

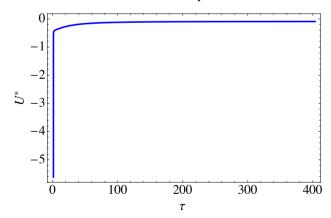


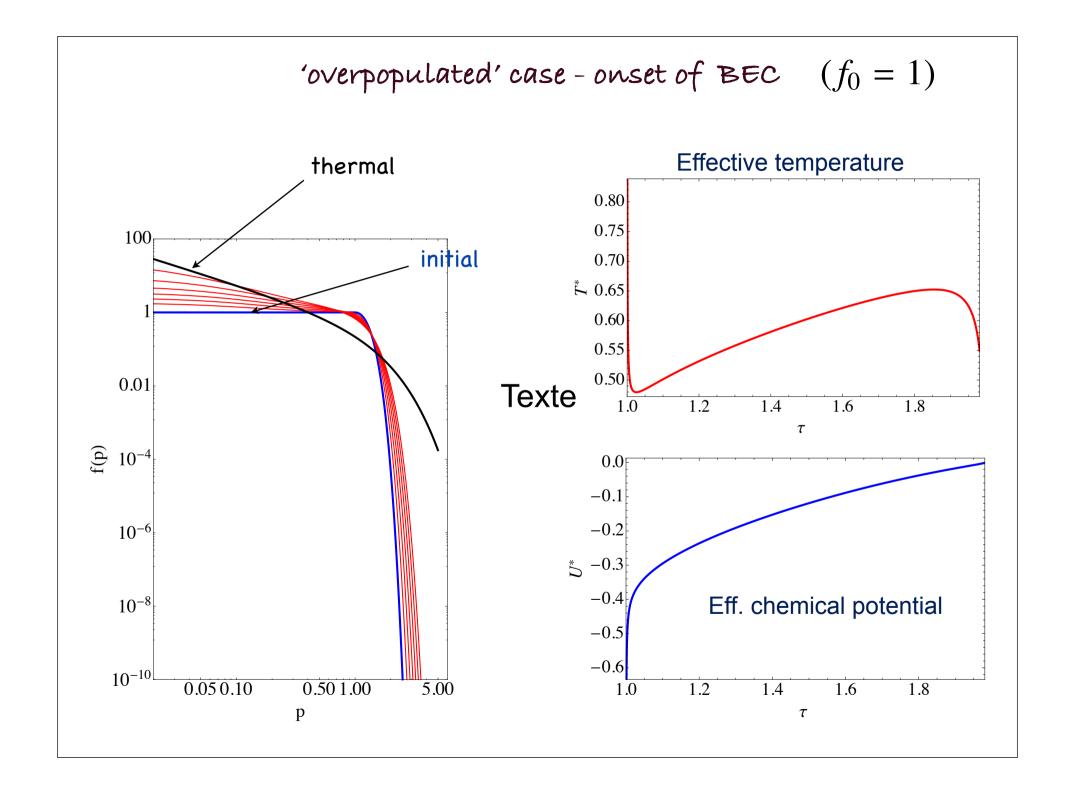






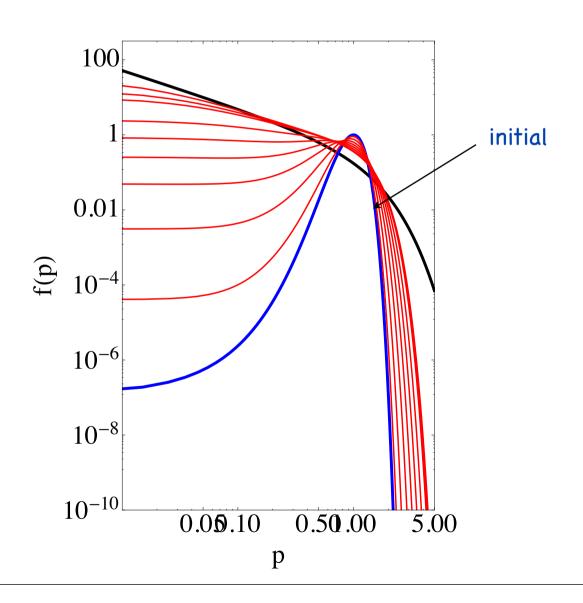
Eff. chemical potential





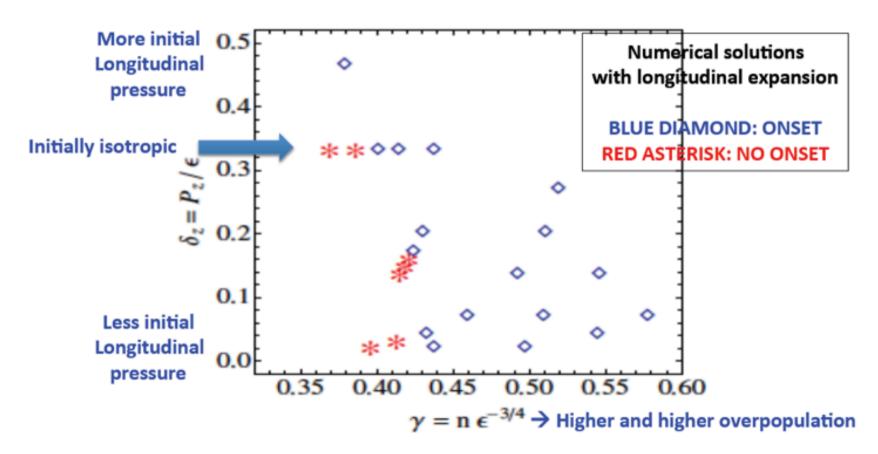
Gaussian initial condition

Same qualitative features



(from Jinfeng Liao, Quark-Matter 2012)

ROBUST AGAINST ANISOTROPY & EXPANSION



High enough initial occupation f_0~1 is able to compete with anisotropy and expansion to reach BEC onset!

Summary

- initial states of colliding heavy nuclei at high energy are characterized by 'overpopulated' gluonic states
- because of the large occupation, the system remains strongly interacting in spite of the small coupling constant
- the (dynamical) growth of (very) soft modes seems to a be a robust feature. It may lead to the formation of a (transient) Bose condensate.